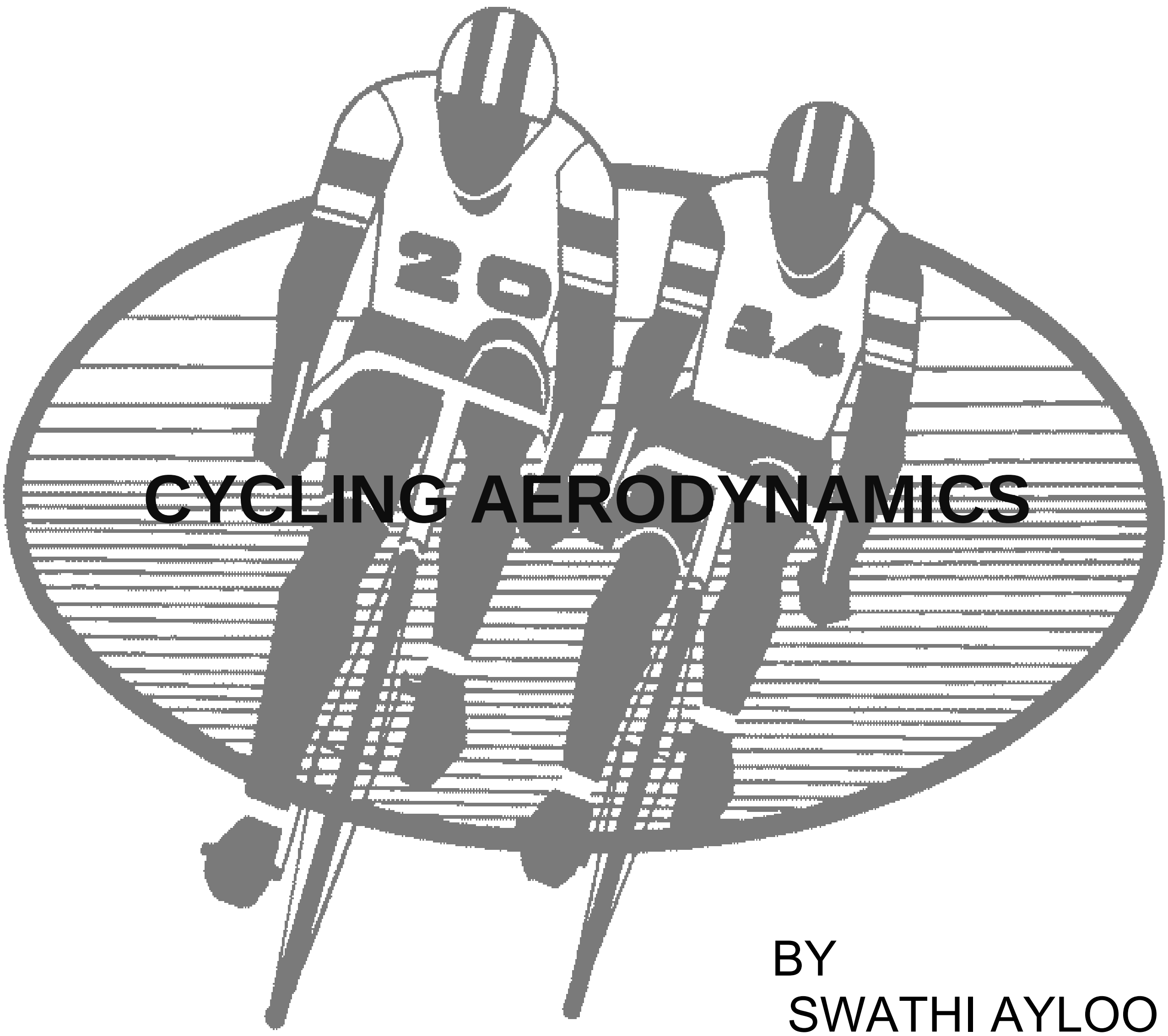
TRANSPORT PHENOMENA & UNIT OPERATIONS

CHOOSE-FOCUS-ANALYZE EXERCISE



CYCLING AERODYNAMICS

BY
SWATHI AYLOO
BT06B027

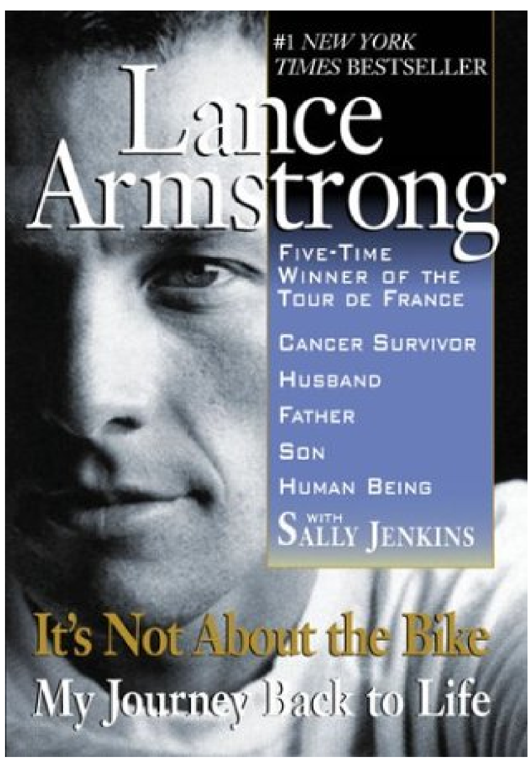
INSPIRATION FOR THE TOPIC

I recently read the book 'It's Not About The Bike: My Journey Back To Life' by Lance Armstrong (international cyclist who won the Tour De France 7 times consecutively post cancer).

To quote him "I think it's fair to say that it's the most physically demanding event in the world. What could be harder?"

Though it is more or less an autobiography, he describes the various aspects of cycling in his book.

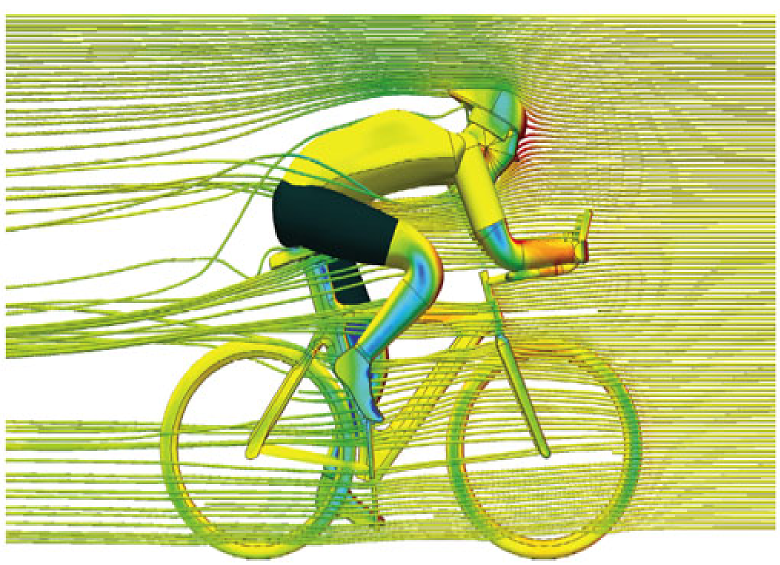
- When the cyclist is riding there are many factors which come into play, one of them being the air drag. A smart cyclist would position himself in such a way so as to minimize the drag as much as possible. Thus, I intend to construct a fluid model, analyze the various parameters which affect it and then estimate the optimum conditions for cycling.
- Fatigue is caused by the production of lactic acid in the body. There is a measure of this in the body by a parameter called 'lactate threshold'. A study can be done on the concentration levels and the rheological factors (fluid flow) of lactic acid in the body.



1. INTRODUCTION

Aerodynamics is the study of how a solid body moves through the air. When we relate this definition to the sport of cycling, it's about how a rider and machine overcome air resistance created by forward motion and the prevailing wind. The speed that can be achieved on the bike is determined by two factors. The first is how much power you are able to produce which is measured in watts and the second is wind resistance, commonly referred to as "drag". The force on an object that resists its motion through a fluid is called drag. When the fluid is a gas like air, it is called aerodynamic drag (or air resistance). When the fluid is a liquid like water it is called hydrodynamic drag. Drag is a complicated phenomenon and is something that is best explored experimentally.

Aerodynamic drag consists of two forces: air pressure drag and direct friction (also known as surface friction or skin friction). A blunt, irregular object disturbs the air flowing around it, forcing the air to separate from the object's surface. Low pressure regions from behind the object result in a pressure drag against the object. With high pressure in the front, and low pressure behind, the cyclist is literally being pulled backwards. Streamlined designs help the air close more smoothly around these bodies and reduce pressure drag. Direct friction occurs when wind comes into contact with the outer surface of the rider and the bicycle. Racing cyclists often wear "skin suits" in order to reduce direct friction. Direct friction is less of a factor than air pressure drag.



2. APPROACH AND CALCULATIONS

We know that $F = PA$, where
P – Pressure
A - Cross sectional area projected in the direction of motion

Using Bernoulli's principle, $P = 0.5\rho v^2$ $F = 0.5\rho A v^2$

However, in case of drag force, the medium also has a role to play and that is accounted by a term C_d called 'coefficient of drag'.

Thus, we now have a standard, simple and compact equation for this drag force:

Drag, $\boxed{D = (0.5)C_d \rho A v^2}$ -------- (1)

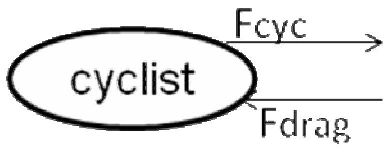
Also, all the terms in this equation can be logically explained as to why they figure out here.

- Drag increases with the <u>density</u> of the fluid (ρ). More density means more mass, which means more inertia, which means more resistance to getting out of the way. The two quantities are directly proportional.
  $D \quad \rho$
- Drag increases with <u>area</u> (A). The greater the area that is in contact with the fluid, the higher is the drag force.
  $D \quad A$
- Drag increases with speed (v). An object that is stationary with respect to the fluid will certainly not experience any drag force. Once it starts moving a resistive force will arise.
  $D \quad v^2$
- Drag is also influenced by the texture, viscosity and compressibility of the fluid. This is where the coefficient of drag of the fluid comes into play.
  Hence $D = (0.5)C_d \rho A v^2$

2.1 VELOCITY PROFILE:

Using equation (1) and balancing forces, the velocity profile of the cyclist can be determined.

$\sum F = ma$



Where, F_{cyc} - force applied by the rider,

F_{drag} - drag force

Calculations:

$F_{cyc} – F_{drag} = ma$

$F_{cyc} – F_{drag} = mdv/dt$

$F_{cyc} – kv^2 = mdv/dt$, where $k = (0.5)C_d\rho A$

$$\int dt = \int m \frac{dv}{Fcyc - kv2}$$

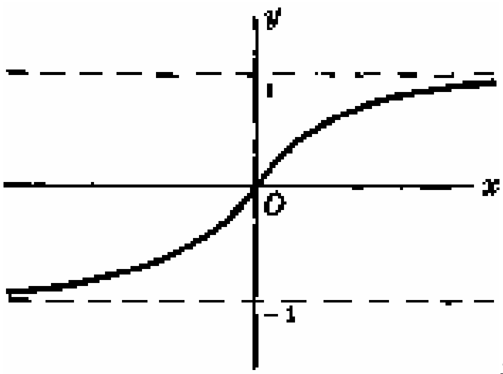
Integrating from 0 to t and 0 to v using integral calculus,

$$t = \frac{m}{\sqrt{kFcyc}} \tanh^{-1}\left(v\sqrt{\frac{k}{Fcyc}}\right) \text{----------} ②$$

$$v = \left(\sqrt{\frac{Fcyc}{k}}\right) \tanh\left(\frac{t}{m}\sqrt{kFcyc}\right) \text{------} ③$$

Thus, velocity is a hyperbolic tangent function(!!) of time, F_{cyc}, and k where $k=(0.5)C_d\rho A$.

This equation brings out the complexity that is involved with drag forces. The terminal velocity can be obtained by substituting $dv/dt = 0$. The above equations also reflect the fact that terminal velocity (v_t) increases with increase in the ratio of mass to area. Hence, small heavy objects have a higher v_t than large light ones.



This is a how a hyperbolic tangent graph looks (y=tanh x). The actual graph of v vs t could not be drawn here because the terms k, m and F_{cyc} vary from one rider to another.

Source: www.math10.com

The actual velocity profile is close to what we just obtained by using simple force balance and basic math. However, equation (1) is applicable only when the range of conditions is small – i.e, there are no large variations in the speed, viscosity, direction of wind and other such factors. Once we intend to extend this to all possible cases then C_d has to be taken as a variable, as a function of certain set of parameters and analysis of the motion becomes complicated.

This is the computation which involves the external forces on the body of the rider, i.e, in the above calculations the cyclist was taken as our system and force balance was done assuming that the drag force is exactly opposite in direction (180^0) to the force applied by the rider. In reality, it is not necessary that the wind always act in that particular direction. It can attack the rider from any angle possible. This is another important factor which determines the cycling performance of the rider.

2.2 ANGLE OF ATTACK:

As mentioned above, this essentially means the direction or the angles in which the rider is attacked by the wind. This is where the rider's position plays an important role in the overall aerodynamics of the bicycle and the rider. In the sports of cycling, it is generally said that the rider accounts for 65-80% of the drag. A typical 70 kg rider on a regular bike with standard wheels will have a drag of about 8 pounds, a better position will reduce drag to about 7 pounds, and an excellent position will yield a drag of 6 pounds.

Le Tour de France is the greatest bicycle race in the world. It covers over 4000 km in three weeks of daily racing. It is a test of rider's speed, strategy and heart. The riders have to travel through straight roads to uphills to down slopes. And for each kind there is a way in which the rider positions himself to improve his performance.

Straight roads Steep hills Mountain climbing

Source: www.efluids.com

2.3 WITHIN THE BODY

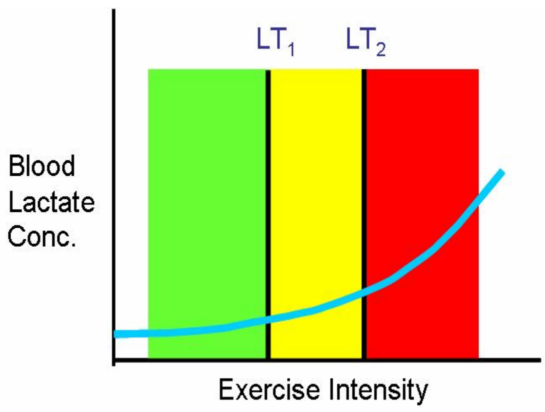
All this while, I have focussed on the external factors which determine the overall performance of the rider. However, there are certain processes that occur in the cyclist's body during his motion which affect his efficiency and performance.

Lactic acid and VO_2 max are the determining parameters and vary from person to person. The next section will be dealing with these two factors and their close dependence through which I obtained certain results.

2.3.1 LACTIC ACID

One such fluid in the body is lactic acid. Fatigue is caused by the production of lactic acid. During power-intensive exercises such as sprinting, when the rate of demand for energy is high, lactate is produced faster than the ability of the tissues to remove it and lactate concentration begins to rise. This is a beneficial process since the regeneration of NAD^+ ensures that energy production is maintained and exercise can continue.

“Lactate threshold” (LT) is a common term used with reference to bicyclists. In physical terms, this is a point on the exercise intensity scale where blood lactate concentration starts to increase. During exercise, lactic acid is being simultaneously produced by working muscles and removed by other muscles as well as the heart, liver, and kidneys. If production rate equals removal rate, then blood lactate concentration will be stable. If production exceeds removal rate, lactate concentration increases.



The green zone represents an exercise intensity range where lactate production is low and lactate removal easily matches production. The yellow zone represents a range of intensities where we see a marked increase in blood lactate prododuction. But, lactate removal also increases so that a new **stable** blood lactate concentration is achieved. Finally, the red zone represents intensities where lactate production now exceeds the maximal rate of blood lactate removal. Exercise in this intensity range results in accumulation of lactate acid and fatigue.

Source: Seiler & Kjerland, Scandinavain Journal of Medicine and Science in Sports

2.3.2 VO_2 max

VO_2 max is volume per time oxygen – maximum. It is the maximum amount of oxygen that can be used by the body for maximum sustained power output(exercise). VO_2 max defines an endurance athlete’s performance ceiling, in other words a person’s “engine”. VO_2 max significantly determines performance in endurance based events such as cycling, triathlon, running, etc.

The factors that affect VO_2 max are altitude, gender and age.

Altitude: An increase in altitude results in a decrease in air density and the partial pressure (or amount) of oxygen in the air.

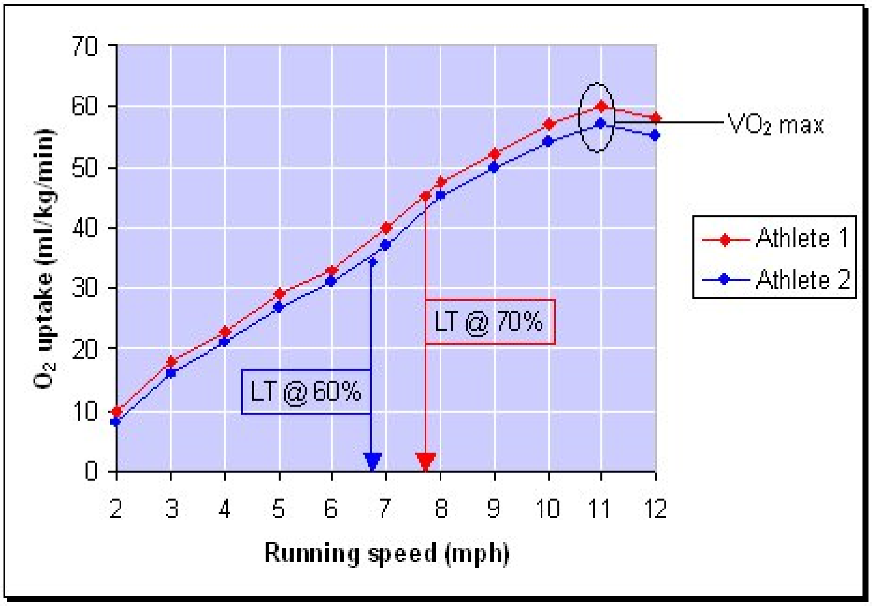
Age: Peak physiological function occurs at about 30 years of age. VO_2max decreases approximately 30 percent between the ages of 20 and 65, with the greatest decline after age 40.

Gender: In general, females tend to have lower VO_2max values than males (15%-20% less). This is primarily due to differences in body composition, haemoglobin content in the blood, and heart size.

2.3.3. LACTIC ACID AND VO_2 max

As mentioned earlier, these two go hand in hand. For instance, untrained individuals usually reach the LT at about 60% of VO_2 max. With training, LT can increase from 60% to above 70% or even higher. Elite endurance athletes and top athletes typically have LTs at or above 80% of VO2 max.

Fact: Three time Tour de France winner Greg LeMond is reported to have had a VO_2 max of 92.5 ml/kg/min at his peak - one of the highest ever recorded. Lance Armstrong's VO_2 max is around 85 ml/kg/min.



Source: www.sport-fitness-advisor.com

Normal values of lactic acid: 0.5-2.2 molar.
VO_2 max of an average young untrained person: 38-45 ml/kg/min.

VO_2 max is measured by measuring the oxygen and carbon dioxide concentration in the inhaled and exhaled air. VO_2 max is reached when oxygen consumption remains at steady state despite an increase in workload.

This can also be seen as a measure of the difference in the arterial and venous oxygen content in the blood i.e, it can be considered as a case of diffusion.

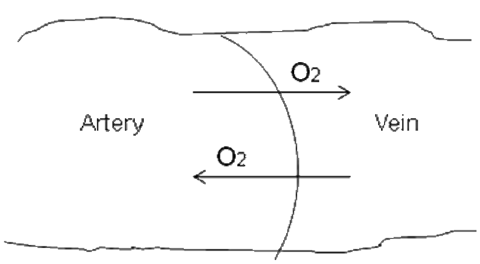
According to Fick's law:

$J= -D *(\partial C/\partial x)$ ---------(4)

Where J – mass flux,

D – Diffusivity,

$\partial C/\partial x$ – concentration gradient.



Model: Here the diffusing substance is oxygen (gas), the two medium being arteries and veins.

Applying the same here,

VO_2 max $(CaO_2 – CvO_2)$

VO_2 max is defined by Fick's law as

$VO_2 \text{ max} = Q(CaO_2 – CvO_2)$--------(5)

where Q is the cardiac output and is defined as
Q = stroke volume(SV)*heart rate(HR)

Stroke Volume is the amount of blood pumped by the right/left ventricle of the heart in one contraction.

Heart rate is the frequency of the cardiac cycle.

Consider the case of a typical **30 year old**, **70 kg** weight untrained person.

Data used:

- In a healthy 70 kg man, medical data show that his end-diastolic volume is about 120 ml and end-systolic volume is 50 ml, giving a stroke volume of 70 ml, i.e , SV = 70 ml.
- The best way to measure the heart rate of a person accurately is by measuring the pulse rate (generally the wrist-radial artery). Theoretically

speaking, there is no “accurate” equation which can be followed to estimate HR. However, the often cited one is

HR = 210 - (0.65 x Age in years) ------------⑥

Hence, HR = 210 – (0.65 x 30) = 190.5 ~ 190

Therefore, Q = (190 x 70)/1000 = 13.3

- $CaO_2 - CvO_2$: This is around 15-25 for any person in normal conditions. A value of 20 can be taken for this.

Therefore, VO_2 max = (13.3 x 20)/100

VO_2 max = 2.66

Now, once VO_2 max is obtained, lactate threshold can be estimated from the graph as it is now known that from VO_2 estimations LT can be estimated.

In reality, this value is around 2.8-3.8 (an **error** of about **11%**)

This shows that we can make changes in the present model and the scope of this model can be discussed which has been done in the conclusion part.

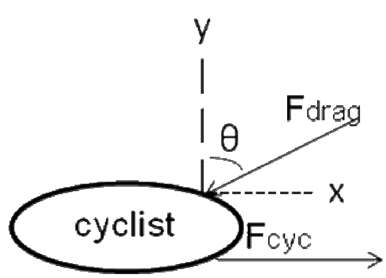
3. ANALYSIS – CONCLUSIONS

Assumptions have resulted in a slight loss of generality.

In the first section:

- The drag force, $D = (0.5)C_d\rho Av^2$ can be used only for short ranges, i.e, no large variations in the speed, viscosity, direction of wind, etc. However, once we want to take into account all these cases too, our system becomes complicated and the drag force will have parameters that in turn will depend on other factors. This can be done incorporating few changes in the terms of the equation and obviously the math part involved can get quite cumbersome.

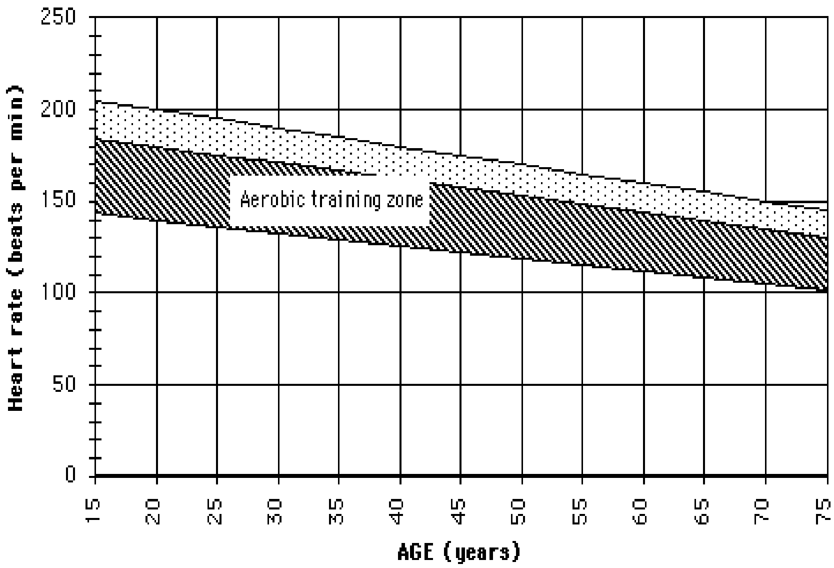
- I took the most simple case wherein the drag force acts anti-parallel to the direction of the rider's force, i.e, essentially, "angles of attack" was not considered in this model of analysis. This can be handled much more easily compared to the previous point, just that the angle of attack can be anything. The only change would be to apply force balances in both the horizontal and vertical directions.



- Another important assumption is regarding F_{cyc}. No person can ride in such a way so as to not have any change in the direction of his motion. Also, in the integration part I assumed F_{cyc} to be constant which is generally not the case. However, in order to avoid complexity in the making up of the system it would be better if we chose to assume it to be constant.

In the second section:

- To start with, the model that I considered is the simplest model possible which definitely is not the case in reality and this has resulted in an error.
- One of the major reasons the error could have occurred is possibly because of the calculations of heart rate, HR. As mentioned earlier, the best way to determine HR would be by measuring the pulse rate of a person which is the simplest thing to do. However, for all theoretical purposes, it becomes imperative to depend on an equation which has been established in medicine.



The graph here shows heart rate as a function of age. Going back to the analysis part, a 30 year old will have a HR between 130-180. The value that I was obtained by calculations was 190!

<u>Source:</u> *www.physionet.org*

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4. REFERENCES

- www.wikipedia.org
- www.efluids.com
- www.sports-fitness-advisor.com
- Fundamentals of Physics by David Halliday, Robert Resnick and Jearl Walker
- “It’s Not About the Bike: My Journey Back to Life” by Lance Armstrong
- Transport Phenomena by Edwin N. Lightfoot

ACKNOWLEDGEMENT

I sincerely thank Prof. G.K.Suraishkumar for providing me this opportunity to work on this exercise. It indeed was a learning experience at every stage. Doing this CFA exercise a second time actually improved my analyzing skills and forced me to think differently.

Lance Armstrong has become my all-time favourite since I've read the book. His book inspired me on and off, not just in choosing this topic but on a personal level too.